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LETTER TO THE EDITOR

Gauge field theory of transport and magnetic relaxation in underdoped cuprates

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Abstract. An interpretation of the transport and magnetic relaxation properties of underdoped cuprates based on the recently proposed $U(1) \times SU(2)$ Chern–Simons gauge field theory is proposed, which takes into account the short-range antiferromagnetic order. The interplay of the doping-dependent spin-gap (explicitly derived by us) effect and dissipation due to gauge fluctuations gives rise to a crossover from metallic to insulating behaviour of the conductivity as temperature decreases, in semi-quantitative agreement with experimental data. For the same reason the magnetic relaxation rate shows a maximum nearby. Various crossover temperatures related to spin-gap effects are shown to be different manifestations of the same energy scale.

Achieving an understanding of the anomalous normal-state properties of oxide superconductors has been a challenge for theorists since their discovery [1]. Recently, a great deal of attention has been focused on underdoped superconductors [2–5], for which pseudogap (spin-gap) effects are fundamental. We will concentrate on the doping range where the short-range antiferromagnetic order (SRAFO) exists [6], and propose an interpretation of the transport and magnetic relaxation properties in this region, based on the recently proposed $U(1) \times SU(2)$ gauge field theory [7].

The linear temperature dependence of the resistivity in most oxide superconductors over a wide range of temperature is well established and a number of explanations have been proposed [8] including the U(1) gauge field theory [9]. On the other hand, in underdoped samples, a resistivity minimum and a crossover from metallic to insulating behaviour have been observed [10–12]. A similar divergence of the resistivity at low temperatures has been found in superconducting samples in strong magnetic fields [13], suppressing the superconductivity. An apparently 'obvious' explanation of these two related phenomena would be localization of charge carriers in two dimensions. However, a more careful comparison of theory with experiments shows [14] that the localization effects including carrier interactions cannot correctly interpret the data. Several other explanations have been proposed based on non-Fermi-liquid (FL) behaviour of charge carriers [15–17], but the zero-field experiments [10–12] have not been addressed, except for in [17] where a gauge field approach was used. We, instead, will concentrate on the latter case. We will show that the presence of SRAFO, leading to a finite mass of spinons (bosons), is the correct starting point in this doping range. The self-generated U(1) holon–spinon (h/s) gauge field becomes singular due to coupling with holons (fermions) [9], which, in turn, renormalizes the massive spinons in a nontrivial way. At low temperatures, effects due to finite spinon mass prevail, leading to insulating behaviour, while at higher temperatures the dissipation caused by the gauge field dominates and gives rise to metallic behaviour. For similar reasons, the spin relaxation rate is low at both low and high temperatures, reaching a maximum near the resistivity crossover point, which is also consistent with experiment [18].

Following a strategy previously applied to the 1D t–J model which reproduced there the known exact Bethe *ansatz* results [19], the Chern–Simons bosonization with the $\mathrm{U}(1) \times \mathrm{SU}(2)$ gauge field [20] was applied to the 2-D t–J model in the limit $t \gg J$, allowing us to rewrite the partition function (and the correlation functions) in terms of a spin-$\frac{1}{2}$ fermion field ψ_α, $\alpha = 1, 2$, minimally coupled to a U(1) field B (gauging global charge), and an SU(2) field V (gauging global spin) whose dynamics is given by a Chern–Simons action [7]. We decomposed the fermion field ψ_α into a product of a spinless fermion field H (holons) and a spin-$\frac{1}{2}$ boson field Σ_α (spinons), satisfying the constraint $\Sigma_\alpha^* \Sigma_\alpha = 1$, thus introducing a local U(1) gauge invariance called h/s. We proved the existence of an upper bound of the partition function for holons in a spinon background, and we found the optimal spinon configuration (an $s + \mathrm{i}\,d$-like RVB state) saturating the upper bound on average. On neglecting the feedback of holon fluctuations to field B and spinon fluctuations to field V, the holon field is a fermion one and the spinon field is a hard-core-boson one. Within this approximation, the 'mean field' (MF) $\bar{B}$ produces a π-flux phase for holons, converting them into Dirac-like fermions, while the $\bar{V}$-field, taking into account the feedback of holons, produces a gap for spinons vanishing in the zero-doping limit.

The continuum action for AF fluctuations around the 'MF', described by a spin-$\frac{1}{2}$ boson field z_α, $\alpha = 1, 2$ (still 'spinons'), is given by [7]

$$S_s = g^{-1} \int \mathrm{d}x_0\, \mathrm{d}^2x\, [v_s^{-2} |(\partial_0 - A_0) z|^2 - |(\partial_\mu - A_\mu) z|^2 + m_s^2 z_\alpha^* z_\alpha] \qquad (1)$$

where A is the h/s gauge field, $g = 8/J$, and $v_s = \sqrt{2} J a$, with a the lattice constant. The spinon 'mass' term $m_s^2 \sim \langle \bar{V}^2 \rangle \sim -\delta \ln \delta$ (the main new feature) is due to averaged perturbation caused by holons of concentration δ via $\bar{V}$. This explicit doping dependence was *derived*, rather than assumed in the theory. It produces a SRAFO, with correlation length $\xi_{AF} \sim (-\delta \ln \delta)^{-1/2}$, fully consistent with the neutron scattering data [21].

Neglecting the gauge fluctuations, holons are described by FL theory with a Fermi surface (FS) consisting of four 'half-pockets' centred at $(\pm\pi/2, \pm\pi/2)$. The MF $\bar{B}$ turns the spinless fermion H into two species of two-component Dirac fermions $\psi^{(r)}$, $r = 1, 2$, each of them being supported on one Néel sublattice. The continuum action for these fermions is given by [7]

$$S_h = \int \mathrm{d}x_0\, \mathrm{d}^2x \sum_r \bar{\psi}^{(r)} [\gamma^0 (\partial_0 - e_r A_0 - \delta) + t (\not{\partial} - e_r \not{A})] \psi^{(r)} \qquad (2)$$

where $\not{A} = \gamma_\mu A_\mu$, $\not{\partial} = \gamma_\mu \partial_\mu$, $\gamma_0 = \sigma_z$, $\gamma_\mu = (\sigma_y, \sigma_x)$, the charges $e_r = \pm 1$ depending on the sublattice. After integrating out the gapful Dirac modes, we end up with a FL-like system of holons with Fermi energy $\epsilon_F \sim t\delta$, interacting through gauge field A.

As shown in [22], for the gauge field model the in-plane resistivity is approximately given by

$$R = \lim_{\omega \to 0} \omega [(\mathrm{Im}\, \Pi_s^\perp(\omega))^{-1} + (\mathrm{Im}\, \Pi_h^\perp(\omega))^{-1}] \qquad (3)$$

where $\Pi_s^\perp$ and $\Pi_h^\perp$ denote the transverse polarization bubbles (at $\vec{q} = 0$) due to the h/s currents of holons and spinons, renormalized by gauge fluctuations. The A-propagator for small $|\vec{q}|$, ω, $\omega/|\vec{q}|$ in the Coulomb gauge is given by [9, 23]

$$\langle A_\mu^\perp A_\nu^\perp \rangle (q, \omega) \sim (\mathrm{i}\omega \lambda_h(\vec{q}) + \chi |\vec{q}|^2)^{-1} \qquad \langle A_0 A_0 \rangle (q, \omega) \sim (\nu_h + \omega_p)^{-1} \qquad (4)$$

where $\lambda_h \sim \kappa/|\vec{q}|$, $\kappa \sim O(\delta)$ is the Landau damping due to the finite FS for holons, $\chi = \chi_h + \chi_s$, $\chi_h \sim m_h^{-1} \sim O(\delta^{-1})$, $\chi_s = v_s m_s^{-1} \sim O((-\delta \ln \delta)^{-1/2})$ is the diamagnetic susceptibility, ν_h is the holon density at the FS, and ω_p is the plasmon gap.

An estimate of the holon contribution to the resistivity can be derived as in [9]:

$$R_h \sim \delta\left[\frac{1}{\epsilon_F \tau_{imp}} + \left(\frac{T}{\epsilon_F}\right)^{4/3}\right] \tag{5}$$

where τ_{imp} is the transport relaxation time due to impurities.

To estimate the spinon contribution, we derive the large-scale behaviour of the spinon current $j^\mu = z^* D_A^\mu z$ correlation function, where $D_A^\mu = \partial_\mu - A_\mu$, by eikonal approximation [24], strictly preserving gauge invariance. We use spinon Green functions at zero temperature, as is partially justified by the spinon gap, but we retain the temperature dependence of the gauge fluctuations.

We apply the Fradkin representation [24] to the spinon propagator

$$\langle z(x) z^*(y)\rangle = G(x, y|A).$$

It can be derived using a first-quantized path integral form of the propagator, with metric $(-++)$, replacing integration over trajectories $q_\mu(t)$ by integration over 3-velocities $\phi_\mu = \dot{q}_\mu(t)$, $\mu = 0, 1, 2$. Rescaling x_0 to $v_s x_0$ one obtains

$$\begin{aligned} G(x, y|A) &= \mathrm{i}\int_0^\infty \mathrm{d}s\, \mathrm{e}^{-\mathrm{i}sm^2}[\mathrm{e}^{\mathrm{i}s(\partial_\mu - A_\mu)^2}](x, y) \\ &\sim \mathrm{i}\int_0^\infty \mathrm{d}s\, \mathrm{e}^{-\mathrm{i}sm^2}\int \mathcal{D}\phi^\mu(t)\, \exp\left(\frac{\mathrm{i}}{4}\int_0^s \phi_\mu^2(t)\,\mathrm{d}t\right)\exp\left(\mathrm{i}\int_0^s \tilde{A}_\mu(t)\phi^\mu(t)\,\mathrm{d}t\right) \\ &\quad\times \int \mathrm{d}^3p\, \exp\left(\mathrm{i}p_\mu\left[x^\mu - y^\mu - \int_0^s \phi^\mu(t)\,\mathrm{d}t\right]\right). \end{aligned} \tag{6}$$

Using an identity (equation (41) from the second paper of [24]), the integral

$$\int_0^s \tilde{A}_\mu \phi^\mu(t)\,\mathrm{d}t \qquad \text{with } \tilde{A}_\mu = A_\mu\left(x + \int_0^t \phi(t')\,\mathrm{d}t'\right)$$

can be decomposed into a sum of an integral along a straight line (denoted by $\int_x^y A$) and a gauge-invariant part depending on the field strength $F_{\mu\nu}$. Thus

$$G(x, y|A) = \exp\left\{-\mathrm{i}\int_x^y A\right\} G(x, y|F).$$

The spinon current-density correlation $\langle j^\mu(x) j^\mu(y)\rangle$, $\mu = 1, 2$, is approximately given by $\langle D_A^\mu(x) G(x, y|A) D_A^\mu(y) G(x, y|-A)\rangle$, where $\langle\cdot\rangle$ denotes averaging w.r.t. A. The gauge-dependent terms of the two spinon propagators exactly cancel each other, yielding a strictly gauge-invariant result, at large scale given approximately by

$$\left\langle \frac{\partial}{\partial x_\mu} G(x, y|F)\, \frac{\partial}{\partial y_\mu} G(x, y|-F)\right\rangle.$$

The $A^\perp$-average involves contributions weighted by the 'magnetic field' correlations $\langle F_{\mu\nu}(z) F_{\rho\sigma}(w)\rangle$, $\mu, \nu, \rho, \sigma = 1, 2$, approximately evaluated for $|z^0 - w^0| \ll T^{-1}$ as in references [9, 23], giving $(\delta_{\mu\rho}\delta_{\nu\sigma} - \delta_{\mu\sigma}\delta_{\nu\rho})(4T/\chi)\mathrm{e}^{-|\vec{z}-\vec{w}|^2 q_0^2} q_0^2$, where $q_0 = (\kappa/[\chi\beta])^{1/3}$ is a momentum cut-off related to the anomalous skin effect due to the Reizer singularity in the $A^\perp$-propagator [9]. The A_0-average involves contributions weighted by 'electric field' correlations $\langle F_{0\mu}(z) F_{0\nu}(w)\rangle$, $\mu, \nu = 1, 2$. Since they vanish in the limit $q, \omega \sim 0$ (see (4)), their contributions will be neglected.

The integrals in (6) can be approximately calculated for relatively low temperatures ($T < \chi m_s^2$) and the current–current correlation becomes

$$\langle j^\mu(x) j^\mu(0)\rangle \sim \left[\frac{\partial}{\partial x_\mu}\exp\left(-\mathrm{i}(x_0^2-|\vec{x}|^2)^{1/2}\left[m^2-\frac{T}{\chi}f(|\vec{x}|q_0/2)\right]^{1/2} - \frac{Tq_0^2}{4\chi}\frac{g(|\vec{x}|q_0/2)}{m^2}(x_0^2-|\vec{x}|^2)\right)(x_0^2-|\vec{x}|^2)^{-1/2}\right]^2 \qquad (7)$$

where, for a real argument, f is monotonically increasing, and vanishing at zero argument, and g is monotonically decreasing, and vanishing at large arguments. Their explicit expressions are lengthy and will be given elsewhere [25]. In deriving the spinon current correlation at $\vec{q} = 0$ we carry out the $\vec{x}$-integration by saddle-point methods. For $x_0 \gg q_0^{-1}$ the integral is dominated by a complex saddle point at $|\vec{x}| = 2q_0^{-1}\alpha(x_0)$, with finite $\alpha(x_0)$ (in the first quadrant), having a weak dependence on x_0. To justify the saddle-point approximation we need to assume that $T > \chi m_s q_0$. It turns out that in the physical range of parameters considered in this letter, this and the above conditions are both satisfied for temperatures between tens of and a few hundred degrees.

Let us define

$$\Pi^+(\omega) = \int_0^\infty \mathrm{d}x_0\, \langle j^\mu j^\mu\rangle(\vec{q}=0, x_0)\mathrm{e}^{\mathrm{i}x_0\omega}.$$

Using the Lehmann representation we find

$$\lim_{\omega\to 0} \mathrm{Im}\,\Pi^\perp(\omega)\omega^{-1} = -2\frac{\partial}{\partial\omega}\mathrm{Re}\,\Pi^+(0).$$

Noting that the main contribution comes from small x^0, introducing a lower cut-off, and performing scale renormalization, we obtain for the $\omega \to 0$ limit

$$\frac{\partial\,\mathrm{Re}\,\Pi^+(\omega)}{\partial\omega} \sim \mathrm{Re}\left[(\alpha(0))^3(q_0)^{3/2}\mathcal{Z}^{1/4}(\mathrm{i}f'')^{-1/2}\left(\frac{T}{\chi}\right)^{-1/2}(\omega-\mathcal{Z}^{1/2})^{-1}\right]$$

where

$$\mathcal{Z} = |\mathcal{Z}|\mathrm{e}^{-\mathrm{i}\theta} \equiv m^2 - \frac{T}{\chi}f(\alpha(0)) \qquad f'' \equiv f''(\alpha(0)) \qquad (8)$$

(renormalization eliminates the contribution of the g-function, being subleading). We find that at large x_0, $\arg\alpha(0) = \pi/4$ and $\arg f''(\alpha(0)) = 0$. We extrapolate $\alpha(x_0)$ to $\alpha(0)$, retaining these features. In this way we recover the correct behaviour, $R \to \infty$, as $T \to 0$. As $x_0 \to 0$, the saddle point extrapolates to $x_s \sim q_0^{-1}\mathrm{e}^{\mathrm{i}\pi/4}$, and we find the 'spinon contribution' to the resistivity:

$$R_s = 2^{-4}\left(\frac{|f''|}{\kappa}\right)^{1/2}|\alpha(0)|^{-3}\frac{|\mathcal{Z}|^{1/4}}{\sin(\theta/4)}. \qquad (9)$$

In figure 1 our calculated resistivity (the sum of (5) and (9)) is plotted as a function of temperature for various dopings in comparison with experimental data taken for LSCO [10] (inset). We have taken $t/J = 3$, $J = 0.1$ eV. Apart from the resistivity scale, there are no other adjustable parameters (similarly for figure 2—see later). We find a resistivity minimum below 100 K in very good agreement with experiment. We see from (8) that the imaginary part of $\mathcal{Z}$ is proportional to temperature T. At low temperatures the spin-gap effect ($\sim m_s$) dominates, $\theta \to 0$, so the system shows an insulating behaviour. (The functional dependence $R \sim 1/T$, different from the 'standard' exponential law due to the spin gap, is a prediction of our theory.) In contrast, at higher temperatures the imaginary and real parts of $\mathcal{Z}$ become comparable, so the

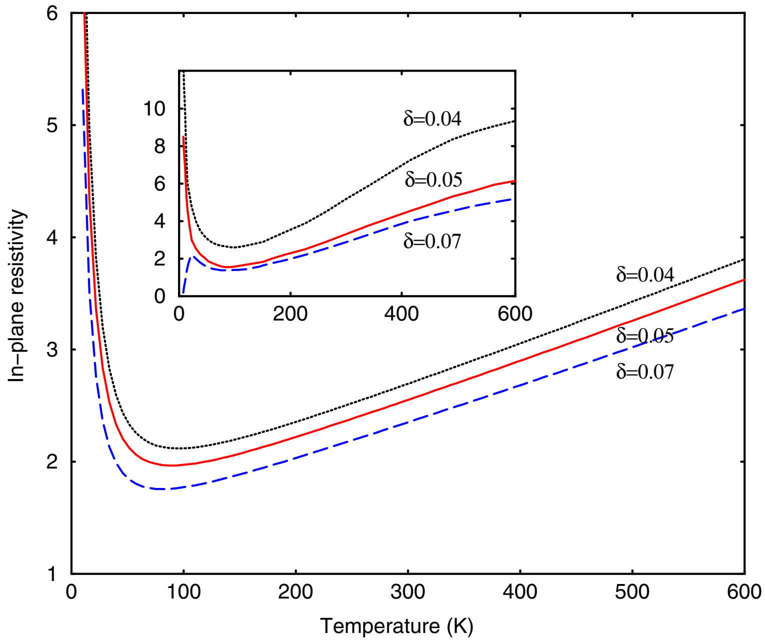

Figure 1. The calculated temperature dependence of the in-plane resistivity (the sum of expressions (5) and (9)) for various dopings δ in comparison with the corresponding experimental data (inset) for $La_{2-\delta}Sr_{\delta}CuO_4$ in units of mΩ cm, taken from [10].

resistivity grows with temperature due to gauge fluctuations via $|\mathcal{Z}|$. Moreover, the minimum shifts to higher temperatures, as the doping decreases, also in agreement with experiment (our theoretical prediction $m_s^2 \sim -\delta \ln \delta$, rather than $\sim\delta$, is responsible for this shift). We have also compared the calculated conductivity on the semi-logarithmic scale with data taken for a very good single crystal of $La_{1.96}Sr_{0.04}CuO_4$ (inset) [6, 11]. We find a symmetric shape of the curve around the maximum, and an inflection point as well as a linear piece on the low-temperature side in both theory and experiment. So far we have not included the external magnetic field. We believe that the experimentally observed crossover from metallic to insulating behaviour in strong magnetic fields when superconductivity is suppressed [13] can be understood in a similar way, and this issue will be addressed in our future communication [25].

Now turn to the spin–lattice relaxation rate T_1^{-1} which can be expressed approximately as [18]

$$(T_1 T)^{-1} \sim \lim_{\omega \to 0} \int \mathrm{d}^2 q \; \mathcal{F}(\vec{q}) \frac{\mathrm{Im}\, \chi_s(\vec{q}, \omega)}{\omega}$$

where χ_s is the spin susceptibility and $\mathcal{F}(\vec{q})$ is the form factor. To evaluate χ_s we use the representation for the spin deduced for large scales:

$$\vec{S}_x \sim \mathrm{e}^{\mathrm{i}\pi|x|} z^* \vec{\sigma} z(x)(1 - \rho_h(x))$$

where ρ_h is the holon density, to be replaced by its average δ. Around the AF wave vector $\vec{\mathcal{Q}}_{AF} = (\pi, \pi)$ we find that

$$\langle \vec{S}(x) \cdot \vec{S}(0) \rangle \sim (1 - \delta)^2 \mathrm{e}^{\mathrm{i}\pi|\vec{x}|} \langle G(x, 0|F) G(x, 0|-F) \rangle$$

which can be calculated as before. Defining

$$\chi^+(\vec{q}, \omega) = \int_0^\infty \mathrm{d}x_0 \; \langle \vec{S} \cdot \vec{S} \rangle (\vec{q}, x_0) \mathrm{e}^{\mathrm{i}x_0\omega}$$

using the Lehmann representation and taking into account that $\mathcal{F}(q)$ is even in $\vec{q}$, one obtains

$$(T_1T)^{-1} = -2\int \mathrm{d}^2q\ \mathcal{F}(\vec{q})\frac{\partial\,\mathrm{Re}\,\chi^+(\vec{q},\omega=0)}{\partial\omega}.$$

Since $\mathcal{F}(\vec{q})$ is peaked around $\vec{Q}_{AF}$ for Cu, integrating over q in a small region around that point, and introducing a cut-off in the real space $\Lambda \sim \pi/|x_s|$, we find

$$(T_1T)^{-1} \sim (1-\delta)^2\sqrt{\delta}|\mathcal{Z}|^{-1/4}\left(a\cos\left(\frac{\theta}{4}\right)+b\sin\left(\frac{\theta}{4}\right)\right) \qquad (10)$$

where

$$a = \mathrm{Re}\int_\Lambda \mathrm{d}^2y\ J_0(2|\vec{y}|\alpha(0)) \qquad b = -\mathrm{Im}\int_\Lambda \mathrm{d}^2y\ J_0(2|\vec{y}|\alpha(0))$$

and J_0 is the zero-order Bessel function.

In figure 2 we plot our calculated spin–lattice relaxation rate $(T_1T)^{-1}$ for ^{63}Cu as a function of temperature for various dopings in comparison with experimental data taken for underdoped samples of YBCO [18]. We observe a maximum near the crossover temperature for conductivity, although the shape around the maximum is not symmetric any longer, due to the presence of the cosine term.

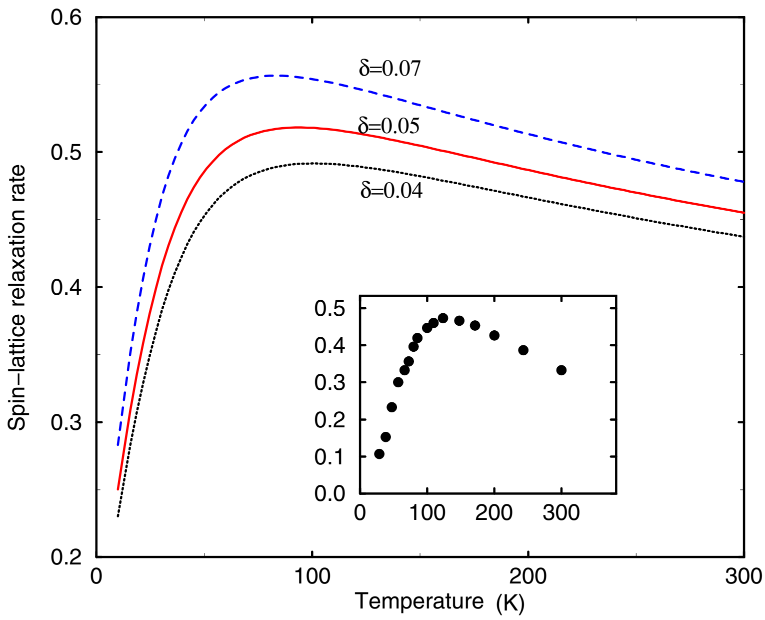

Figure 2. The temperature dependence of the calculated spin–lattice relaxation rate $(T_1T)^{-1}$ given by (10). Inset: $^{63}(T_1T)^{-1}$ in the CuO_2 planes of $YBa_2Cu_3O_{6.52}$ single crystals in units of $s^{-1}\ K^{-1}$, taken from [18].

To summarize, we have shown using the $U(1)\times SU(2)$ gauge field theory that the metal–insulator behaviour crossover and peculiar behaviour of the NMR relaxation in underdoped cuprates might be due to the interplay of the spin-gap (derived in our approach) effect and gauge field fluctuations. More precisely, the crossover takes place when the real and imaginary parts of $\mathcal{Z}$ (equation (8)) become comparable. In figure 3 we have plotted three different crossover temperatures related to the spin-gap effects, namely the metal–insulator crossover T_{M-I} (the minimum of the in-plane resistivity), and the spin-gap crossover temperatures, detected by

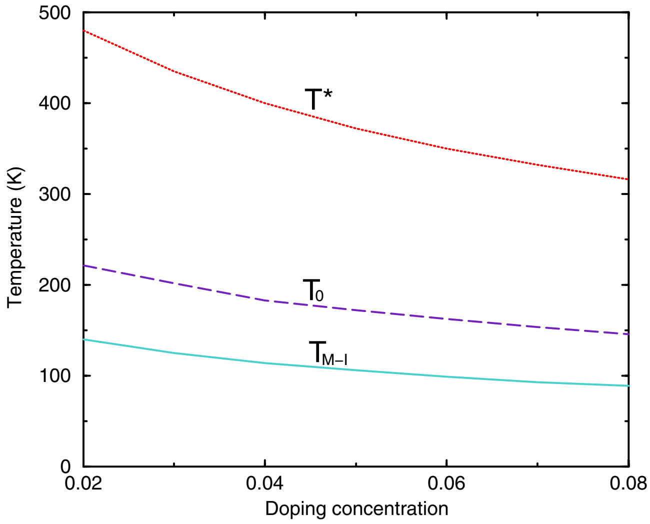

Figure 3. The calculated metal–insulator crossover temperature T_{M-I}, the inflection point T_0 in the NMR $(T_1T)^{-1}$, and the inflection point T^* of R as functions of doping.

means of NMR (T_0) and by means of resistivity (T^*), identified with their respective inflection points, in the low-doping region ($\delta \sim 0.02$–0.08). The latter two temperatures, roughly speaking, limit from above the region of significant spin-gap effects and validity of our approximation. These crossover temperatures are different manifestations of the same energy scale. The fact that their relative order $T^* > T_0 > T_{M-I}$, as well as their order of magnitude, agrees with experiments provides further support for our approach.

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